Calculations on the effect of stylus drag on a turntable platter

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1 Introduction

Please note that this document is a work in progress, and errors might exist
Feel free to leave a comment or two if you find an error, or want to contribute with something.

2 Calculating deceleration caused by stylus drag

2.1 Platter moment of inertia

This assumes that the platter is solid, and made from one material. My platter is made from three layers of MDF board, with a steel hub. Since the diameter of the hub is relatively small, the higher density of it will have very little effect on the moment of inertia.

r_p : Radius of the platter in meters (m)
m_p : Mass of the platter in kilograms (kg)

$$I = \frac{1}{2} * m_p * r_p^2 \tag{1}$$

2.2 "Mass" seen at a distance from center

To continue we need to know the "mass" as seen by a force applied at a distance r_f from the center of the platter.
m_f : The "mass" to be found
r_f : Distance from the center of the platter for which the "mass" will be calculated

$$I = m_f * r_f^2 \Rightarrow m_f = \frac{I}{r_f^2} \tag{2}$$

If we insert the result from equation 1...

$$m_f = \frac{m_p * r_p^2}{2 * r_f^2} \tag{3}$$

2.3 Acceleration if a force is applied

What will the acceleration/deceleration of the platter be if a force (i.e. stylus drag) is applied to it at a distance r_f from the center of the platter?

Suppose the circle with radius r_f is straightened out to a line.
According to Newtons second law of motion, the acceleration is
F : The force applied, in Newtons (N)
a : The acceleration ($\frac{m}{s^2}$)

$$a = \frac{F}{m_f} \tag{4}$$

If we insert equation 3...

$$a = \frac{2 * r_f^2 * F}{m_p * r_p^2} \tag{5}$$

2.4 If a force is applied for a certain period of time...

Δt : The period of time the force is applied in seconds (s)
ΔV : The change of speed in meters/second (m/s)

$$\Delta V = a * \Delta t \tag{6}$$

2.5 Getting the change in RPM

Now we have the change of speed, of an object in linear motion. To make some sense, we need to "convert" it back to revolutions per minute (RPM).
C : Circumference at r_f in meters (m)

$$C = 2 * r_f * \pi \tag{7}$$

The change of rotational speed would then be:
ΔR_{ps} : Revolutions per second
ΔR_{pm} : Revolutions per minute

$$\Delta R_{ps} = \frac{\Delta V}{2 * r_f * \pi} \Rightarrow \Delta R_{pm} = \frac{\Delta V * 60}{2 * r_f * \pi} \tag{8}$$

2.6 Putting it all together

Equations 1, 3, 5 and 8 gives us

$$\Delta R_{pm} = \frac{r_f * F * \Delta t * 60}{m_p * r_p^2 * \pi} \tag{9}$$

Summary of the variables:
ΔR_{pm} : The change in speed, in revolutions per minute.
Δt : The period of time that the increased force is present, in seconds.
F : The force applied to the platter, or increase in force when going from one part of the record to another, in Newtons.
r_f : The distance from the rotational center of the platter where the force is applied, in meters.
m_p : The mass of the platter, in kilograms.
r_p^2 : The radius of the platter, in meters.

3 Examples

3.1 Example 1

What effect on speed stability would increased stylus drag when going from a relatively quiet part to a more dynamic/loud part of the record have?

Newton's first law of motion tells us that the sum of all the forces acting on an object moving at a constant velocity equals zero.
On a turntable, the three most significant forces acting on the platter would be (correct me if I'm wrong):

1. Motor drive
2. Stylus drag
3. Bearing friction

Where the motor represents the force that drives that platter and the second two the counteracting force, that slows it down.

The situation mentioned above (the second law of motion) would be exactly the same if the platter is not rotating (constant velocity of 0): the forces acting on it cancels each other out.

Consider the following:
The platter is not rotating. A force equal to the increase in stylus drag when entering a more dynamic part of the record is applied to the platter.
I have not at this moment had the chance to measure this force, so the value below is quite random.
$m_p = 3$ - Platter mass in kg
$r_p = 0.15$ - Platter radius in meters
$F = 0.001$ - Increase in stylus drag in N ($0.001N = 0.1$) grams
$r_f = 0.1$ - Distance from the center where the force is applied, in meters
$\Delta t = 1$ - Period of time the increase occurs, in seconds (s)

Putting this into equation 9 gives us the change in rotational speed:

$$\Delta R_{pm} = \frac{0.1 * 0.001 * 1 * 60}{3 * 0.15^2 * \pi} = 0.0283 \tag{10}$$

which is a change of $\frac{0.0283}{33\frac{1}{3}} = 0.000849\%$ for a $33\frac{1}{3}$ rpm record

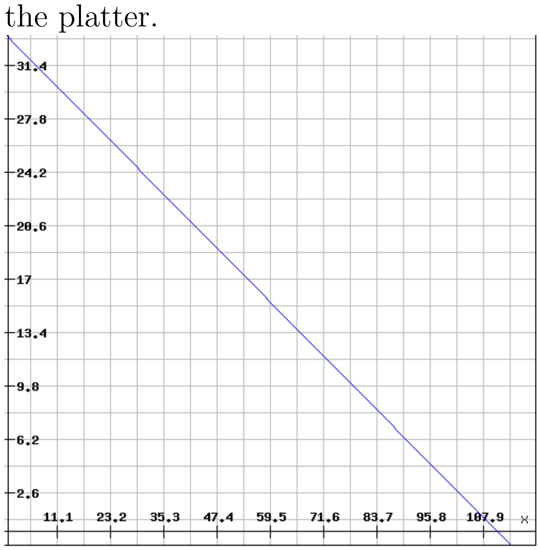
4 Measuring bearing friction and stylus drag

4.1 Bearing friction

By measuring the deceleration of the platter with the drive belt and stylus removed the friction force of the bearing can be calculated.
To measure the deceleration we need to follow these steps.

1. Remove the drive belt
2. Put the tonearm outside the platter
3. Have some way of measuring the speed of the platter. Preferably with quite high resolution and update frequency

Since the friction force acts as a torque on the platter, the result of this measurement will be in Newton meters (Nm). The friction force at any distance from the rotational center can then be calculated from this.
The graph below shows rotational speed (y axis) vs time (x axis). The graph should be quite linear. Otherwise we have some non constant force acting on the platter.



The acceleration (negative deceleration) is given by dividing the change in speed by the period of time: $a = \frac{\Delta s}{\Delta t}$

a_b : The acceleration that the bearing friction causes (m/s^2). Again, deceleration is defined as negative acceleration.
s_{rpm} : The rotational speed in RPM.
s_{ms} : The speed at distance r from the center, in meters per second (m/s)

t : The time in seconds (t)
Converting revolution per minute (RPM) to meters per second (m/s):

$$s_{ms} = \frac{s_{rpm} * 2 * r * \pi}{60} \tag{11}$$

The acceleration is then calculated as:

$$a_b = \frac{\Delta s_{rpm} * 2 * r * \pi}{60 * \Delta t} \tag{12}$$

We already know that $F = m * a$. We also know that $m = \frac{I}{r^2}$ (I=Moment of inertia, which we already calculated in equation 1)
Putting the two equations above together gives us:

$$F = \frac{I * a}{r^2} \tag{13}$$

Inserting the acceleration (a_b) gives us:

$$F = \frac{I * \Delta s_{rpm} * 2 * r * \pi}{60 * \Delta t * r^2} = \frac{I * \Delta s_{rpm} * \pi}{30 * \Delta t * r}(N) \tag{14}$$

Torque is defined as force (in Newtons) times distance (in meters): $\tau = F * r$
Putting equation 14 into this gives us:

$$\tau = \frac{I * \Delta s_{rpm} * \pi}{30 * \Delta t}(Nm) \tag{15}$$

5 more to come...

As soon as I have built equipment to be able to measure the rotational speed of the turntable platter with high accuracy, I will update this with some practical examples.